which the polarization is a maximum. We have, from (25), (22), and (24),

$$d\chi_1=(\sin^2\theta\cos\sigma+\sin^2\sigma\cos\theta)\,d\omega=-2(m-1)\cos\sigma\sin^2\theta\,d\omega,$$

and $\cos\sigma$ is negative. Hence $\chi_1$ decreases as $\omega$ (and therefore $i$) decreases, or as $m$ increases. For $m=1$, $i=\frac{\pi}{2}$ and $\chi_1=\mu^{-2}$; for $m=\infty$, $\cos\sigma=0$, and therefore $\chi_1=0$, or the maximum polarization tends indefinitely to become perfect as the number of plates is indefinitely increased.

For a given number of plates the angle of maximum polarization may be readily found from (24) by the method of trial and error. But for merely examining the progress of the functions, instead of tabulating $i$ for assumed values of $m$, it will serve equally well to tabulate $m$ for assumed values of $i$. The following Table gives for assumed angles of incidence, decreasing by 5° from 90°, the number of plates required to make these angles the angles of maximum polarization of the transmitted light, and the value of $\chi_1$, which determines the defect of polarization.

| $i=$ | 90° | 85° | 80° | 75° | 70° | 65° | 60° | 56° 40′ $(=\varpi)$ |
|---|---|---|---|---|---|---|---|---|
| $m=$ | 1 | 1·330 | 1·944 | 2·913 | 4·921 | 9·775 | 30·372 | $\infty$ |
| $\chi_1=$ | ·433 | ·422 | ·390 | ·337 | ·265 | ·177 | ·075 | 0 |

## V. “On the Theory of the Polyhedra.” By the Rev. T. P. Kirkman, M.A., F.R.S.

This is a revised version of a Paper having the same title, read on the 30th of May, 1861, of which an abstract has been already given at page 218.

## *January* 30, 1862.

Major-General SABINE, R.A., President in the Chair.

The following communications were read:—

## I. “On the Calculus of Symbols.”—Second Memoir. By W. H. L. Russell, Esq., A.B. Communicated by Arthur Cayley, Esq. Received January 7, 1862.

(Abstract.)

This memoir is the continuation of one on the calculus of symbols which I had the honour to lay before the Society in December 1860, and which has since been published in the 'Philosophical Transactions.' I commence this paper with some extensions of the method given in the former memoir for resolving functions of non-commutative symbols into binomial factors. I then explain a method, analogous to the process for extracting the square root in ordinary algebra, for resolving such functions into equal factors. I next investigate a process for finding the highest common internal divisor of two functions of non-commutative symbols, or, in other words, of finding if two linear differential equations admit of a common solution. After this, I give a rule for multiplying linear factors of non-commutative symbols, analogous to the ordinary algebraical rule for linear algebraical factors. I then resume the consideration of the binomial theorem explained in the former memoir. Two new forms of this binomial theorem are here given; and the method by which these forms are proved identical will, I hope, be considered an interesting portion of symbolical algebra, and as exhibiting in a remarkable manner its peculiar nature.

## II. "On Internal and External Division in the Calculus of Symbols." By William Spottiswoode, Esq., M.A., F.R.S. Received January 8, 1862.

(Abstract).

Continuing my researches in the calculus of symbols, I have been led to investigate the most general case of division, viz. that wherein a function of any degree $n$ in $\pi$ is divided, (1) internally, (2) externally, by another function of any other degree $m$ in $\pi$. The investigations here subjoined give (1) the various terms of the quotient, together with their laws of derivation both by actual division and otherwise; (2) the final remainder, and thence the conditions that the divisor may be a factor, internal or external as the case may be, of the dividend. An example has been added in each case by way of illustrating the processes. A remarkable reciprocal relation subsisting between the functions ($\Phi$), of the coefficients ($\phi$) of the dividend, and the corresponding functions ($\Psi$) of the coefficients ($\psi$) of the divisor is exhibited, at the end of the paper.